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K-One Modulo Three Mean Labeling Of Some Path Related Graphs

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Abstract- Jayanthi and Maheswari introduced one modulo three mean labeling and proved some standard graphs are one modulo three mean graphs. We extend this concept to *k*-one modulo three mean labeling. In this paper we discuss *k*-one modulo three mean labeling of some path related graphs.

Keywords- Mean labeling, Mean graphs, k-one modulo three mean labeling, k-one modulo three mean graphs.
AMS Subject Classification— 05C78.

1. INTRODUCTION

This paper deals with graph labeling. All graphs considered here are simple, finite and undirected. The terms not defined here are used in the sense of Harary [7].

Somasundaram and Ponraj [9] have introduced the notion of mean labeling of a (p,q) graph. Different kinds of mean labeling are studied by Gayathri and Gopi in [6]. Swaminathan and Sekar [10] introduced the concept of one modulo three graceful labeling. As an analogue, Jayanthi and Maheswari [8] introduced one modulo three mean labeling and proved that some standard graphs are one modulo three mean graphs.

In [1], we obtained some necessary conditions and properties for one modulo three mean labeling. In [2], we obtained one modulo three mean labeling of some family of trees. In [3], we obtained one modulo three mean labeling of some special graphs. In [4], we obtained one modulo three mean labeling of disconnected graphs. In [5], *k*-one modulo three mean labeling of snake related graphs was obtained. Here we discuss *k*-one modulo three mean labeling of some path related graphs.

2. MAIN RESULTS

Definition 2.1

A graph $G=(p,q)$ is said to be ***k*-one modulo three mean graph** if there is a function f from the vertex set of G to the set $\{0,\ 1,\ 3,\ 4,\ 6,\ 7,\ ...\ 3(k+q)-6,\ 3(k+q)-5\}$ with f is one-one and f induces a bijection $f(x)\in\{4,10,16,\ldots,3k-8\}$ from the edge set of G to the set $=\dfrac{3(k+q)}{6}$ where $f^*(uv)=\left\lceil\dfrac{f(u)+f(v)}{2}\right\rceil$ and the function f is called as ***k*-one modulo three mean labeling** of G. Here, $f^*(uv)\equiv 1 (\text{mod}\, 3)$ for every edge uv in G.

Theorem 2.2

The path P_n is a *k*-one modulo three mean graph for

1) n is even and $k \geq 1$
2) n is odd and $k \geq 2$.

Proof

Let $\{u_i,\ 1 \leq i \leq n\}$ be the vertices and $\{e_i,\ 1 \leq i \leq n-1\}$ be the edges which are denoted as in Figure 2.1.

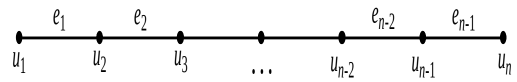

Figure 2.1: Ordinary labeling of P_n

First we label the vertices as follows:
Define $f : V \to \{0, 1, 3, 4, ..., 3(k+q)-6, 3(k+q)-5\}$ by

Case 1: n even and $k \geq 1$

For $1 \leq i \leq n$,

$$f(u_i) = \begin{cases} 3i+3k-6 & i \text{ odd} \\ 3i+3k-8 & i \text{ even} \end{cases}$$

Case 2: n odd and $k \geq 2$

For $1 \leq i \leq n$,

$$f(u_i) = \begin{cases} 3i+3k-8 & i \text{ odd} \\ 3i+3k-6 & i \text{ even} \end{cases}$$

Then the induced edge labels of the both cases are:
For $1 \leq i \leq n-1$,

$$f^*(e_i) = 3i + 3k - 5$$

The above defined f provides *k*-one modulo three mean labeling of the graph P_n.
7-one modulo three mean labeling of the graph P_8 is shown in Figure 2.2.

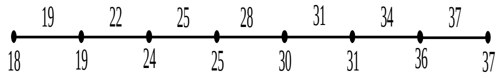

Figure 2.2: 7-OMTML of P_8

Theorem 2.3

The star graph $K_{1,n}$ is a k-one modulo three mean graph for any $k \geq n$.

Proof

Let $\{u_i, 1 \leq i \leq n\}$ be the vertices and $\{e_i, 1 \leq i \leq n\}$ be the edges which are denoted as in Figure 2.3.

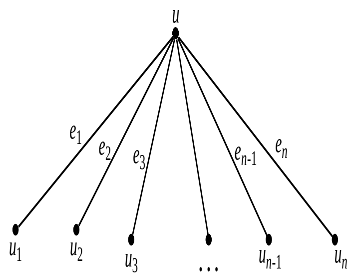

Figure 2.3: Ordinary labeling of $K_{1,n}$

First we label the vertices as follows:
Define $f : V \rightarrow \{0, 1, 3, 4, ..., 3(k + q) - 6, 3(k + q) - 5\}$ by

$$f(u) = 3k + 3n - 5$$

For $1 \leq i \leq n$,

$$f(u_i) = 6(i - 1) + 3(k - n)$$

Then the induced edge labels are:
For $1 \leq i \leq n$,

$$f^*(e_i) = 3(i - 1) + 3k - 2$$

The above defined function f provides k-one modulo three mean labeling of the graph $K_{1,n}$.
16- one modulo three mean labeling of the graph $K_{1,10}$ is shown in Figure 2.4.

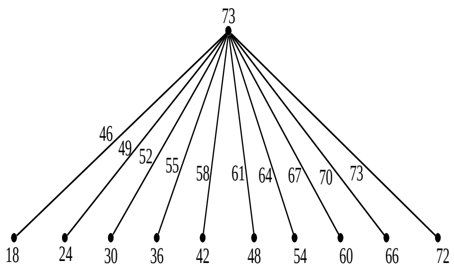

Figure 2.4: 16-OMTML of $K_{1,10}$

Theorem 2.4

The comb graph P_m^+ is a k-one modulo three mean graph for all m and k.

Proof

Let $\{u_i, v_i, 1 \leq i \leq m\}$ be the vertices and $\{e_i, 1 \leq i \leq 2m - 1\}$ be the edges which are denoted as in Figure 2.5.

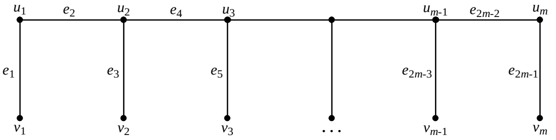

Figure 2.5: Ordinary labeling of P_m^+

First we label the vertices as follows:
Define $f : V \rightarrow \{0, 1, 3, 4, ..., 3(k + q) - 6, 3(k + q) - 5\}$ by
For $1 \leq i \leq m$,

$$f(u_i) = \begin{cases} 6(i-1)+3k-3 & i \text{ odd} \\ 6(i-2)+3k+4 & i \text{ even} \end{cases}$$

$$f(v_i) = \begin{cases} 6(i-1)+3k-2 & i \text{ odd} \\ 6(i-2)+3k+3 & i \text{ even} \end{cases}$$

Then the induced edge labels are:

$$f^*(e_i) = 3i + 3k - 5, \quad 1 \leq i \leq 2m - 1$$

The above defined function f provides k-one modulo three mean labeling of the graph P_m^+.

21-one modulo three mean labeling of the graph P_{10}^+ is shown in Figure 2.6.

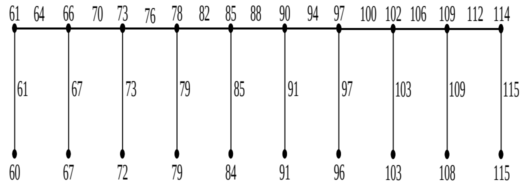

Figure 2.6: 21-OMTML of P_{10}^+

Theorem 2.5

The caterpillar G obtained by attaching n pendant edges to each vertex of the path P_m is a k-one modulo three mean graph for

i) m is even, for all n and $k \geq 1$.
ii) m is odd, for all n and $k \geq n$.

Proof

Let G be a caterpillar obtained from the path P_m by attaching n pendant edges to each of its vertices. Let $\{u_{ij}, u_j, 1 \leq i \leq n, 1 \leq j \leq m\}$ be the vertices and $\{e_{ij}, 1 \leq i \leq n, 1 \leq j \leq m, e_j, 1 \leq j \leq m - 1\}$ be the edges which are denoted as in Figure 2.7.

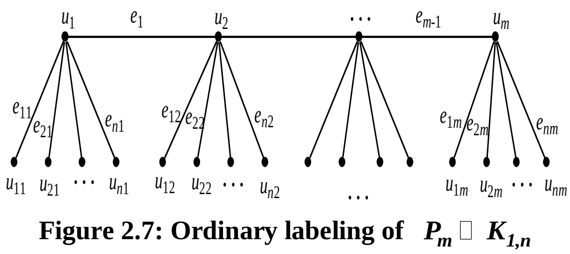

Figure 2.7: Ordinary labeling of $P_m \odot K_{1,n}$

First we label the vertices as follows:
Define $f : V \rightarrow \{0, 1, 3, 4, ..., 3(k + q) - 6, 3(k + q) - 5\}$ by

Case 1: m even, for all n and $k \geq 1$

For $1 \leq i \leq n$, $1 \leq j \leq m$,

$$f(u_{ij}) = \begin{cases} 3(n+1)(j-1)+6i+3k-9 & j \text{ odd} \\ 3(n+1)(j-2)+6i+3k-2 & j \text{ even} \end{cases}$$

$$f(u_j) = \begin{cases} 3(n+1)(j-1)+3k-2 & j \text{ odd} \\ 3(n+1)(j-2)+6n+3k-3 & j \text{ even} \end{cases}$$

Case 2: m odd, for all n and $k \geq n$
For $1 \leq i \leq n$, $1 \leq j \leq m$

$$f(u_{ij}) = \begin{cases} 3(n+1)(j-1)+6(i-1)+3k-3n & j \text{ odd} \\ 3(n+1)(j-2)+6i+3k+3n-5 & j \text{ even} \end{cases}$$

$$f(u_j) = \begin{cases} 3(n+1)(j-1)+3k+3n-5 & j \text{ odd} \\ 3(n+1)(j-2)+3k+3n & j \text{ even} \end{cases}$$

Then the induced edge labels of the both cases are:
For $1 \leq i \leq n$, $1 \leq j \leq m$,
$f^*(e_{ij}) = 3(n+1)(j-1) + 3i + 3k - 5$
For $1 \leq j \leq m - 1$,
$f^*(e_j) = 3(n+1)(j-1) + 3k + 3n - 2$

The above defined function f provides k-one modulo three mean labeling of the graph $P_m \odot K_{1,n}$.
11-one modulo three mean labeling of the graph $P_5 \odot K_{1,4}$ is shown in Figure 2.8.

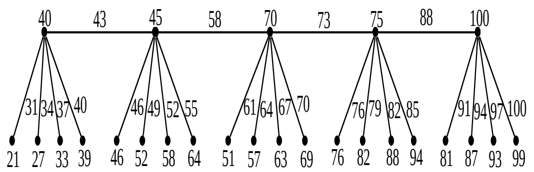

Figure 2.8: 11-OMTML of $P_5 \odot K_{1,4}$

Theorem 2.6

The bistar $B_{m,n}$ is a k-one modulo three mean graph for
1) $m = n$ and $k \geq 1$.
2) $m > n$ and $k \geq m - n + 1$.

Proof

Let $\{u_i, 1 \leq i \leq m, v_i, 1 \leq i \leq n\}$ be the vertices and $\{e_i, 1 \leq i \leq m + n + 1\}$ be the edges which are denoted as in Figure 2.9.

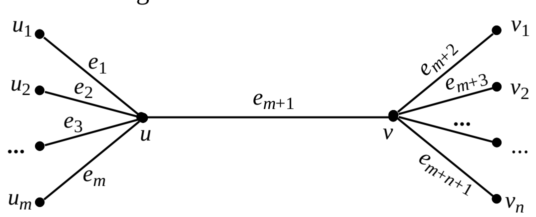

Figure 2.9: Ordinary labeling of $B_{m,n}$

First we label the vertices as follows:
Define $f : V \to \{0, 1, 3, 4, ..., 3(k + q) - 6, 3(k + q) - 5\}$ by

Case 1: $m = n$ and $k \geq 1$
For $1 \leq i \leq m$,
$f(u_i) = 6i + 3k - 8$
For $1 \leq i \leq n$,
$f(v_i) = 6i + 3k - 3$
$f(u) = 3k - 3;$
$f(v) = 3m + 3n + 3k - 2$

Case 2: $m > n$ and $k \geq m - n + 1$
For $1 \leq i \leq m$, $f(u_i) = 6i + 3n - 3m + 3k - 8$
For $1 \leq i \leq n$,
$f(v_i) = 6i + 3m - 3n + 3k - 3$
$f(u) = 3k + 3m - 3n - 3$
$f(v) = 3k + 3m + 3n - 2$
Then the induced edge labels are:
$f^*(e_i) = 3i + 3k - 5 \qquad 1 \leq i \leq m + n + 1$

The above defined function f provides k-one modulo three mean labeling of the graph $B_{m,n}$.
5-one modulo three mean labeling of the graph $B_{8,4}$ is shown in Figure 2.10.

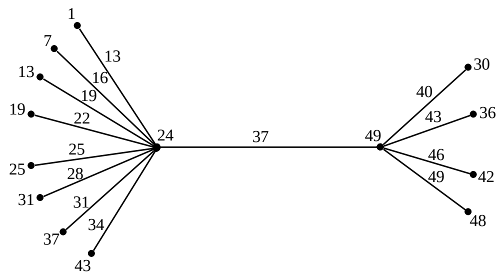

Figure 2.10: 5-OMTML of $B_{8,4}$

Theorem 2.7

The ladder graph $L_n = P_n \times P_2$ is a k-one modulo three mean graph for
1) n is odd and $k \geq 1$
2) n is even and $k \geq 2$.

Proof

Let $\{u_i, u_i', 1 \leq i \leq n\}$ be the vertices and $\{e_i, 1 \leq i \leq 3n - 2\}$ be the edges which are denoted as in Figure 2.11.

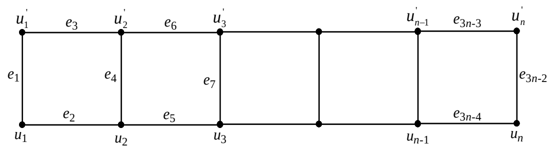

Figure 2.11: Ordinary labeling of L_n

First we label the vertices as follows:
Define $f : V \to \{0, 1, 3, 4, ..., 3(k + q) - 6, 3(k + q) - 5\}$ by

Case 1: n odd and $k \geq 1$
For $1 \leq i \leq n$,

$$f(u_i) = \begin{cases} 9i+3k-12 & i \text{ odd} \\ 9i+3k-14 & i \text{ even} \end{cases}$$

$$f(u_i') = \begin{cases} 9i+3k-11 & i \text{ odd} \\ 9i+3k-9 & i \text{ even} \end{cases}$$

Case 2: n even and $k \geq 2$

For $1 \le i \le n$,

$$f(u_i) = \begin{cases} 9i + 3k - 14 & i \text{ odd} \\ 9i + 3k - 12 & i \text{ even} \end{cases}$$

$$f(u_i^{'}) = \begin{cases} 9i + 3k - 9 & i \text{ odd} \\ 9i + 3k - 11 & i \text{ even} \end{cases}$$

Then the induced edge labels are:
For $1 \le i \le 3n - 2$,

$$f^*(e_i) = 3i + 3k - 5$$

The above defined function f provides k-one modulo three mean labeling of the graph L_n.
6-one modulo three mean labeling of the graph L_8 is shown in Figure 2.12.

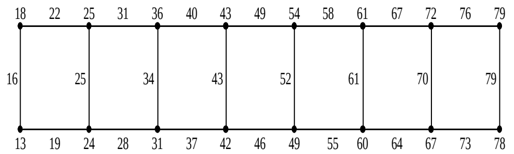

Figure 2.12: 6-OMTML of L_8

Definition 2.8

Let $G = P_m(+)\bar{K}_n$ be the graph with the vertex set $V(G) = \{u_i, v_i, 1 \le i \le m, 1 \le j \le n\}$ and the edge set $E(G) = \{u_iu_{i+1}, u_1v_j, u_mv_j, 1 \le i \le m - 1, 1 \le j \le n\}$.

Theorem 2.9

The graph $G = P_m(+)\bar{K}_n$ is a k-one modulo three mean graph if $m \equiv 0 \pmod 4$ for all $k \ge 1$ and for all n.

Proof

Let $\{u_i, 1 \le i \le m, v_j, 1 \le j \le n\}$ be the vertices and $\{e_i, 1 \le i \le m - 1, a_j, b_j, 1 \le j \le n\}$ be the edges which are denoted as in Figure 2.13.

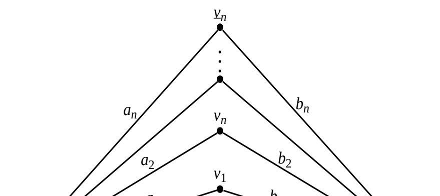

Figure 2.13: Ordinary labeling of $P_m(+)\bar{K}_n$

First we label the vertices as follows:
Define $f : V \to \{0, 1, 3, 4, ..., 3(k + q) - 6, 3(k + q) - 5\}$ by

$$f(u_i) = \begin{cases} 3i + 3k - 6 & 1 \le i \le \frac{m}{2},\ i \text{ odd} \\ 3i + 3k - 5 & \frac{m}{2} + 1 \le i \le m,\ i \text{ odd} \\ 3i + 3k - 8 & 1 \le i \le \frac{m}{2},\ i \text{ even} \\ 3i + 6n + 3k - 9 & \frac{m}{2} + 1 \le i \le m,\ i \text{ even} \end{cases}$$

$$f(v_i) = 6i + 3m + 3k - 8 \quad 1 \le i \le n$$

Then the induced edge labels are:

$$f^*(e_i) = \begin{cases} 3i + 3k - 5 & 1 \le i \le \frac{m}{2} \\ 3i + 3n + 3k - 5 & \frac{m}{2} + 1 \le i \le m - 1 \end{cases}$$

$$f^*(a_j) = 3j + 3\left(\frac{m}{2}\right) + 3k - 5 \qquad 1 \le j \le n$$

$$f^*(b_j) = 3j + 3(m + n) + 3k - 8 \qquad 1 \le j \le n$$

The above defined function f provides k-one modulo three mean labeling of the graph $P_m(+)\bar{K}_n$.
13-one modulo three mean labeling of the graph $P_4(+)\ \bar{K}_7$ is shown in Figure 2.14.

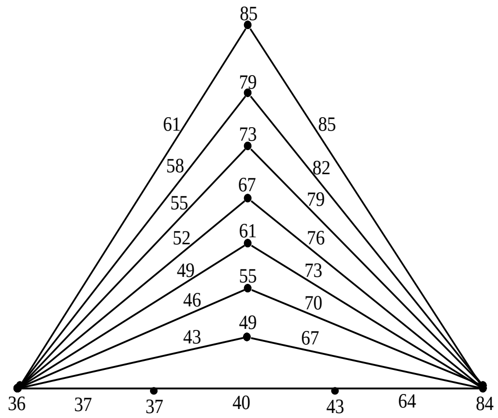

Figure 2.14: 13-OMTML of $P_4(+)\bar{K}_7$

Theorem 2.10

The book graph $K_{1,n} \times P_2$ is a k-one modulo three mean graph if n is even.

Proof

Let $\{u, v, w, u_i, v_i: 1 \le i \le n\}$ be the vertices and $\{a_i, b_i, c_i, 1 \le i \le n\}$ be the edges which are denoted as in Figure 2.15.

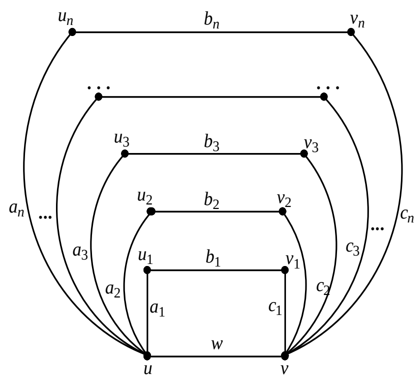

Figure 2.15: Ordinary labeling of $K_{1,n} \times P_2$

First we label the vertices as follows:
Define $f : V \to \{0, 1, 3, 4, ..., 3(k + q) - 6, 3(k + q) - 5\}$ by

$$f(u) = 3k - 3, \quad f(v) = 9n + 3k - 2$$

$f(u_i) = 6i + 3k - 8 \qquad 1 \le i \le n$

Case 1: $n \equiv 0 \pmod 4$

$$f(v_i) = \begin{cases} 6n+6i+3k-8 & 1 \le i \le \frac{n}{4} \\ 6n+6i+3k-2 & \frac{n}{4}+1 \le i \le \frac{n-2}{2} \\ 6i+3k-3 & \frac{n+2}{2} \le i \le \frac{3n}{4} \\ 6i+3k-9 & \frac{3n+8}{4} \le i \le n \end{cases}$$

$$f\left(v_{\frac{n}{2}}\right) = 9n + 3k - 3$$

$$f\left(v_{\frac{3n+4}{4}}\right) = \frac{15n}{2} + 3k - 2$$

Case 2: $n \equiv 2 \pmod 4$

$$f(v_i) = \begin{cases} 6n+6i+3k-8 & 1 \le i \le \frac{n+2}{4} \\ 6n+6i+3k-2 & \frac{n+6}{4} \le i \le \frac{n-2}{2} \\ 6i+3k-3 & \frac{n+2}{2} \le i \le \frac{3n-2}{4} \\ 6i+3k-9 & \frac{3n+6}{4} \le i \le n \end{cases}$$

$$f\left(v_{\frac{n}{2}}\right) = 9n + 3k - 3$$

$$f\left(v_{\frac{3n+2}{4}}\right) = \frac{15n}{2} + 3k + 1$$

Then the induced edge labels are:

$f^*(a_i) = 3i + 3k - 5 \qquad 1 \le i \le n$

$f^*(w) = \frac{9n+6k-4}{2}$

Case 1: $n \equiv 0 \pmod 4$

$$f^*(b_i) = \begin{cases} 3n+6i+3k-8 & 1 \le i \le \frac{n}{4} \\ 3n+6i+3k-5 & \frac{n}{4}+1 \le i \le \frac{n-2}{2} \\ 6i+3k-5 & \frac{n+2}{2} \le i \le \frac{3n}{4} \\ 6i+3k-8 & \frac{3n+8}{4} \le i \le n \end{cases}$$

$$f^*\left(b_{\frac{n}{2}}\right) = 6n + 3k - 5$$

$$f^*\left(b_{\frac{3n+4}{4}}\right) = 6n + 3k - 2$$

$$f^*(c_i) = \begin{cases} \frac{15n}{2}+3i+3k-5 & 1 \le i \le \frac{n}{4} \\ \frac{15n}{2}+3i+3k-2 & \frac{n}{4}+1 \le i \le \frac{n-2}{2} \\ \frac{9n}{2}+3i+3k-2 & \frac{n+2}{2} \le i \le \frac{3n}{4} \\ \frac{9n}{2}+3i+3k-5 & \frac{3n+8}{4} \le i \le n \end{cases}$$

$$f^*\left(c_{\frac{n}{2}}\right) = 9n + 3k - 2$$

$$f^*\left(c_{\frac{3n+4}{4}}\right) = \frac{33n}{4} + 3k - 2$$

Case 2: $n \equiv 2 \pmod 4$

$$f^*(b_i) = \begin{cases} 3n+6i+3k-8 & 1 \le i \le \frac{n+2}{4} \\ 3n+6i+3k-5 & \frac{n+6}{4} \le i \le \frac{n-2}{2} \\ 6i+3k-5 & \frac{n+2}{2} \le i \le \frac{3n-2}{4} \\ 6i+3k-8 & \frac{3n+6}{4} \le i \le n \end{cases}$$

$$f^*\left(b_{\frac{n}{2}}\right) = 6n + 3k - 5$$

$$f^*\left(b_{\frac{3n+2}{4}}\right) = 6n + 3k - 2$$

$$f^*(c_i) = \begin{cases} \frac{15n}{2}+3i+3k-5 & 1 \le i \le \frac{n+2}{4} \\ \frac{15n}{2}+3i+3k-2 & \frac{n+6}{4} \le i \le \frac{n-2}{2} \\ \frac{9n}{2}+3i+3k-2 & \frac{n+2}{2} \le i \le \frac{3n-2}{4} \\ \frac{9n}{2}+3i+3k-5 & \frac{3n+6}{4} \le i \le n \end{cases}$$

$$f^*\left(c_{\frac{n}{2}}\right) = 9n + 3k - 2$$

$$f^*\left(c_{\frac{3n+2}{4}}\right) = \frac{33n+12k}{4}$$

The above defined function f provides k-one modulo three mean labeling of the graph $K_{1,n} \times P_2$.
10-one modulo three mean labeling of the graph $K_{1,6} \times P_2$ is shown in Figure 2.16.

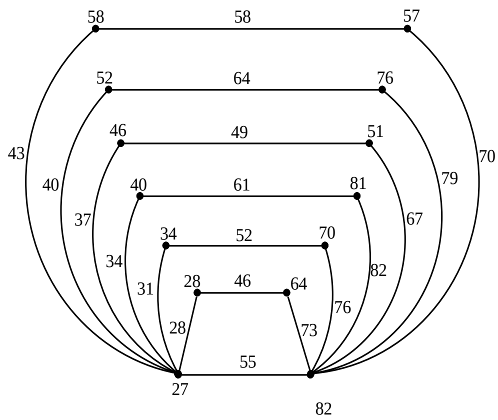

Figure 2.16: 10-OMTML of $K_{1,6} \times P_2$

3. CONCLUSION

Here *k*-one modulo three mean labeling of some path related graphs has been disused. More such graphs can be labeled so that we can find properties on it which will be our future study.

REFERENCES

[1] B. Gayathri and V. Prakash , Necessary conditions for one modulo three mean labeling of graphs, International Journal of Engineering Science, Advanced Computing and Bio-Technology, Vol. 6, No. 2, April-June 2015, pp. 41-61.

[2] B. Gayathri and V.Prakash , One modulo three mean labeling of some family of trees, Accepted for publication in International Journal of Engineering Science, Advanced Computing and Bio-Technology.

[3] B. Gayathri and V. Prakash, One modulo three mean labeling of some special graphs, Mathematical Sciences International Research Journal, Vol. 4, Issue 2(2015), pp. 434 -440.

[4] B. Gayathri and V. Prakash, One modulo three mean labeling of disconnected graphs, Jamal Academic Journal: Interdisciplinary, Special Issue, February 2016, pp. 21-26.

[5] .B. Gayathri and V. Prakash, *k*-One modulo three mean labeling of snake related graphs, International Journal of Scientific Research and Review, Vol. 7, Issue 9(2018), pp. 169 -177.

[6] R. Gopi, A Study on Different kinds of Mean labeling, Ph.D. Thesis, Bharathidasan University, Trichy, February (2013).

[7] F. Harary, Graph Theory, Addison Wesley Reading Massachusetts, 1972.

[8] P. Jeyanthi and A. Maheswari, One Modulo Three Mean Labeling of Graphs, Amer. J. Appl. Math. and Stat., 2(5) (2014) 302-306.

[9] S. Somasundaram and R. Ponraj, Mean labelings of graphs, National Academy Science Letter, 26 (2003) 10-13.

[10] V. Swaminathan and C. Sekar, Modulo three graceful graphs, Proceed. National Conference on Mathematical and Computational Models, PSG College of Technology, Coimbatore, 2001, 281-286.